Two-Dimensional Flow inside MHD Ducts with Transverse Asymmetries

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Theme

DESIGNERS of MHD generator channels, in particular of the diagonally connected type, with appreciable Hall currents J_x (of the order of 1 A/cm^2) and with appreciable diameters (of the order of 1 m) must give special attention to the transverse flow asymmetries that develop inside the channel due to the pressure gradient normal to the electrode walls that is induced by the presence of Hall currents. In the present work, the compressible turbulent flow of seeded plasma in MHD channels is analyzed by a two-dimensional formulation that solves the equations of motion (including the y-momentum equation) by an implicit finite difference scheme. The numerical method solves for all the unknowns, including the gasdynamic pressure and pressure gradient, across the channel at every numerical step taken in the streamwise direction. The present work will show that the Hall current density component J_x, if sufficiently large, can cause asymmetries in the mean gas velocity profile and other gasdynamic profiles.

Contents

Most previous work on the gasdynamic behavior of an MHD channel[1,2] is based on the formulation of a two-dimensional boundary-layer solution along the walls and a quasi-one-dimensional solution in the core of the flow. However, for large-size MHD power generators with Hall currents of the order of 1 A/cm^2, asymmetrical behavior will develop. Such transverse nonuniformities in the gasdynamic distributions affect the current distribution in the channel and therefore the internal resistance, voltage drops, total power and efficiency of the device.

The governing equations in the present formulation, expressed in the Cartesian coordinate system with the usual boundary-layer and eddy viscosity assumptions, are the following:[3]

Continuity:

$$\frac{\partial}{\partial x}(\rho u a)+\frac{\partial}{\partial y}(\rho v a)=0$$

x-Momentum:

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+J_y B+\frac{\partial}{\partial y}\left[(\mu+\varepsilon)\frac{\partial u}{\partial y}\right]$$

y-Momentum:

$$0=\frac{\partial p}{\partial y}+J_x B$$

where $a(x)$ is the effective width of the channel in the z-direction (i.e., after accounting for the boundary layers on the sidewalls of the channel). The rest of the symbols used are self-explanatory and are listed together with the rest of the equations in Ref. 3. The y-momentum equation has been written in its most simplified form after carrying out an order of magnitude analysis of the terms in the full form of the equation.

The numerical procedure is similar to those used to solve the usual boundary-layer equation, that is, starting with some initial profiles and known boundary conditions at the two walls, the computations can proceed in the streamwise direction. This process can apply in general for any type of flow inside the channel, subsonic or supersonic, provided no shocks form inside. If the computations do not satisfy the exit conditions for the subsonic case, this means that the inlet conditions have to be modified.

The boundary conditions for the x-momentum equation are known at each wall (namely $u=0$). The y-momentum and the over-all continuity equations are both first-order equations, each requiring only one boundary condition. However, two boundary conditions exist for the continuity equation (u/v = slope of the channel walls) while none is known for the pressure p in the y-momentum equation. Therefore, an iteration procedure has been developed which utilizes the conservation of the total mass flow rate as the convergence criterion. Details of such iteration is described in Ref. 3.

Figure 1 represents the results obtained for the Avco Mark VI channel[3] using the present method. The figure shows the variation of the mean gas velocity, temperature, and pressure in the core of the flow (on the centerline) under three different loading conditions. The numerical results agree reasonably well with the results obtained at Avco. However, when the current density

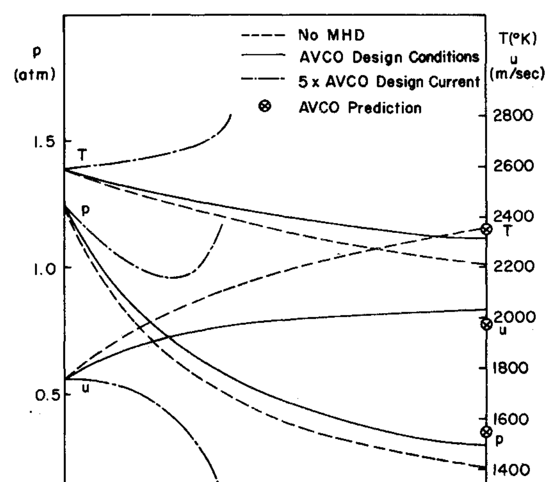

Fig. 1 Computed variations of gas velocity u, temperature T, and pressure p along the centerline of the Avco Mark VI channel, for three different operating conditions (fixing the inlet conditions).

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Index categories: Boundary Layers and Convective Heat Transfer–Turbulent; Nozzle and Channel Flow; Plasma Dynamics and MHD.

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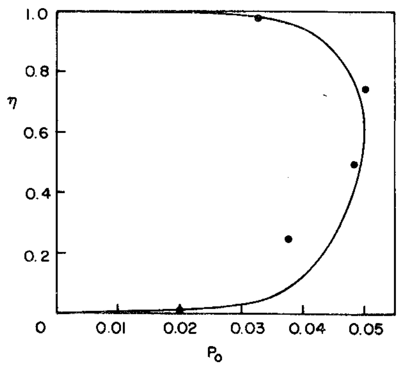

Fig. 2 Normalized pitot-pressure profiles for the Hirho accelerator at $x = 0.31$ m (solid line shows the computed profile, the points indicate the experimental measurements). Bottom line is the anode wall.

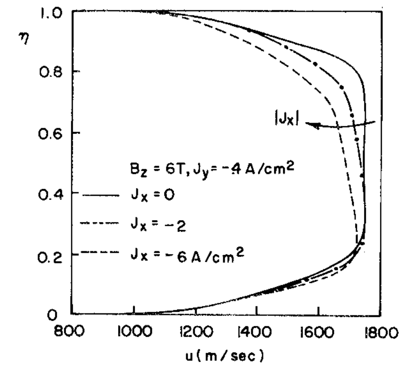

Fig. 4 Effect of the Hall current on the velocity profiles $u(\eta)$. ($x = 1.1$ m). The bottom line is the cathode wall.

level is increased, the retarding $J_y B$ force not only slows the mean gas velocity but leads to the separation of the boundary layer.

Although this application was for the Faraday mode where the Hall current is negligible, it is presented here to show the validity of the procedure also when the normal pressure gradient is practically zero.

The Hirho accelerator[4] is considered as a good application of our new method because of the extremely high current densities J_x and J_y along the channel [$|J_x| > 25.0$, $J_y > 40.0\ A/\mathrm{cm}^2$]. Figure 2 is a plot of the pitot-pressure profile across the channel at $x = 0.31$ m from the entrance of the channel. It shows qualitatively good agreement with the existing experimental measurements. It also illustrates the nonuniformities and asymmetries that develop across the channel because of the Hall current effects: the maximum value in the profile occurs towards the cathode wall (of the accelerator).

A hypothetical application also is considered in which the current density level is allowed to vary. The channel inlet conditions and geometry are taken similar to those of the Avco Mark VI channel except for the width which is kept constant. First, the Hall current is kept constant at a high level (4.0 A/cm^2) and the loading (J_y) is increased until the retarding force $J_y B_z$

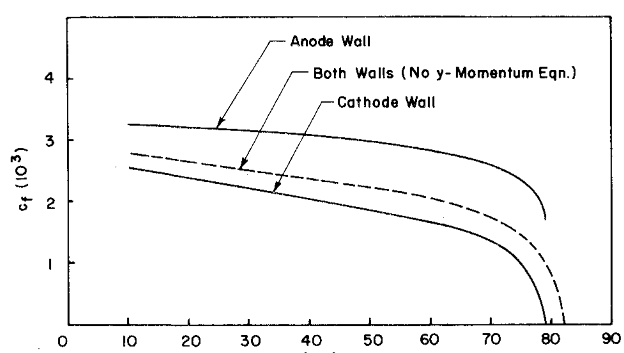

Fig. 3 Skin friction variation along the channel walls with and without the inclusion of the y-momentum equation ($B_z = 6T$, $J_x = -4$, $J_y = -1.6\ A/\mathrm{cm}^2$).

eventually caused the boundary layer to separate. Figure 3 presents the variation of the skin friction coefficient along the two electrode walls of the channel with and without considering the y-momentum equation (effect of Hall current). When considering the y-momentum equation, the pressure (hence the density) will be lower near the cathode wall (J_x is assumed flowing upstream) and, accordingly, the low density layers of the flow will respond much quicker to the retarding force. Therefore, the boundary layer is expected to separate first on the cathode wall.

To dramatize the effect of the Hall current on the asymmetry of the flow for the same application, J_y is kept constant at a low level ($-0.4\ A/\mathrm{cm}^2$) while J_x is allowed to vary. Figure 4 shows the variation of the mean gas velocity across the channel at $x = 1.1$ m for different J_x. It is clear that the asymmetry in the profile increases as $|J_x|$ increases. Also, the velocity profile is much fuller near the cathode wall than near the anode wall. Once again, this is primarily due to the difference in density between the boundary layers on the two walls because of the influence of the Hall current J_x.

Although the values of the Hall currents considered above are beyond the normal limits of MHD generator applications, one has to bear in mind that in the last application the height of the channel is 18 cm. In real life MHD channels, the height will be in the order of 1 m and therefore even with realistic values of Hall currents ($\sim 1.0\ A/\mathrm{cm}^2$) the transverse asymmetries are expected to be of the order presented here.

References

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